

18. konferenca
Dnevi slovenske informatike
**Poenostavljana
Lagrangeova
dinamika odprtih
kinematskih verig**

Lado Lenart , Jan Babič

18. 04. 2011

LINEARNI IN KOTNI MOMENT

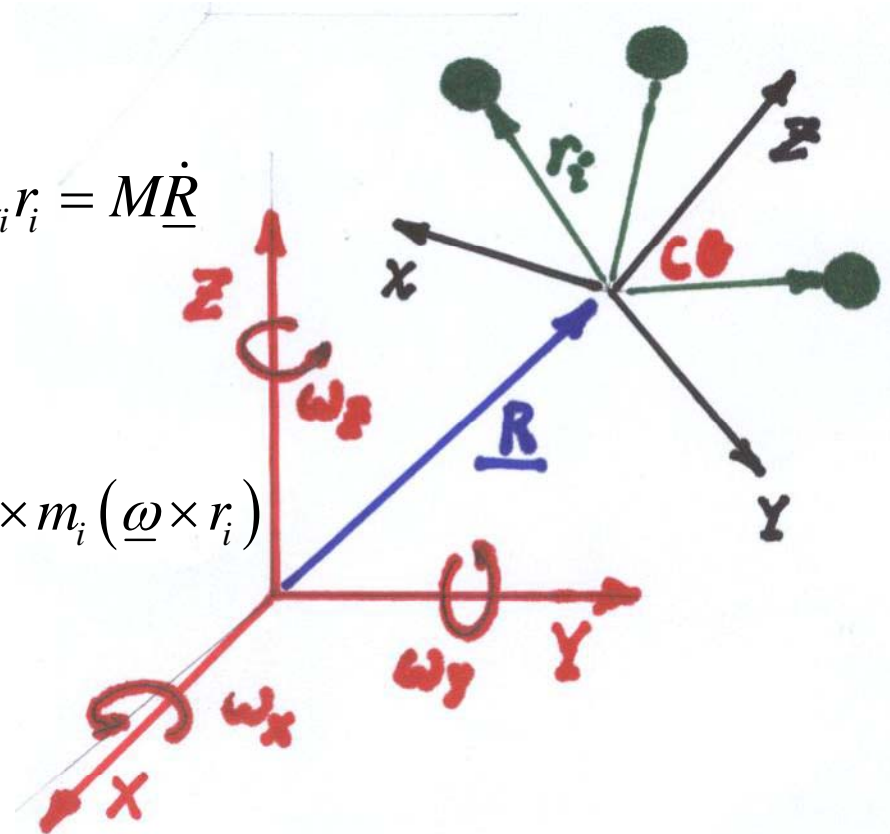
$$\frac{d}{dt}(\underline{R} + r_i) = \underline{\dot{R}} + \underline{\omega} \times r_i$$

$$\sum m_i (\underline{\dot{R}} + \underline{\omega} \times r_i) = \underline{\dot{R}} \sum m_i + \underline{\omega} \times \sum m_i r_i = M \underline{\dot{R}}$$

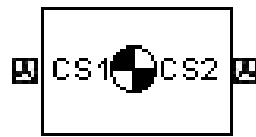
$$\sum (\underline{R} + r_i) \times m_i (\underline{\dot{R}} + \underline{\omega} \times r_i) = \underline{R} \times M \underline{\dot{R}} + \sum r_i \times m_i (\underline{\omega} \times r_i)$$

$$\frac{d}{dt}(M \underline{\dot{R}}) = \underline{F}$$

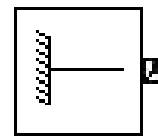
$$\frac{d}{dt} \left[\sum r_i \times m_i (\underline{\omega} \times r_i) \right] = \underline{T}$$



Uporabljeni elementi ‘SIMULINK’, ‘SimMechanics’



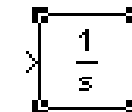
Body



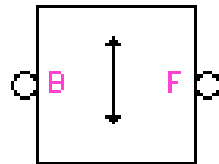
Ground



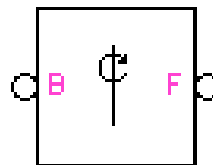
Machine Environment



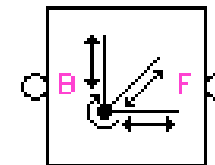
Integrator



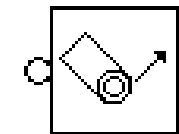
Prismatic



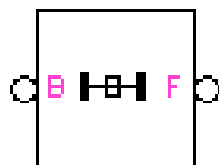
Revolute



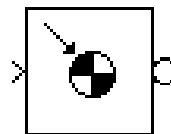
Six-DoF



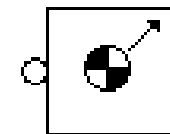
Joint Sensor



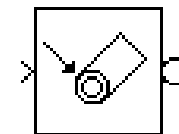
Weld



Body Actuator

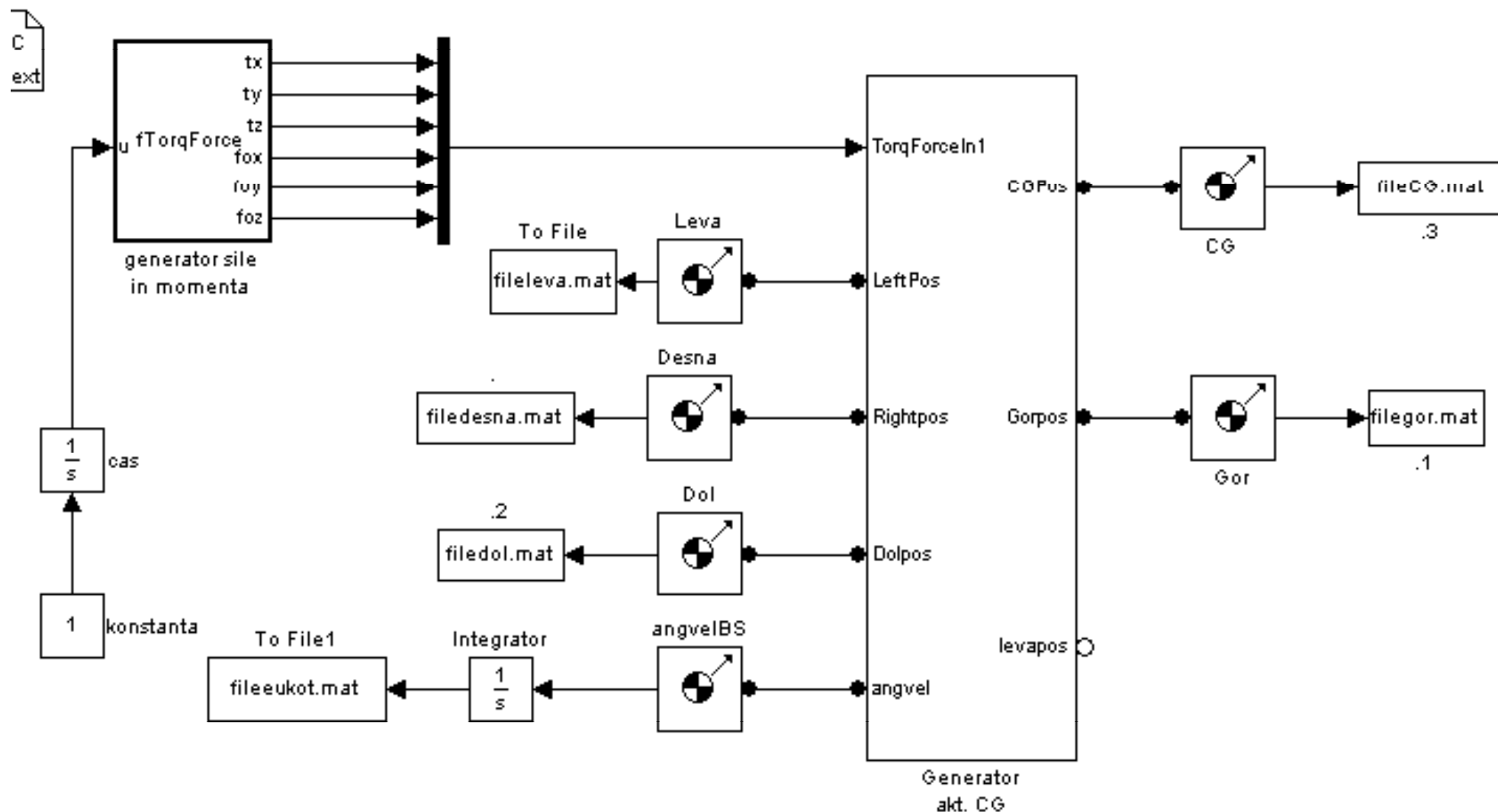


Body Sensor

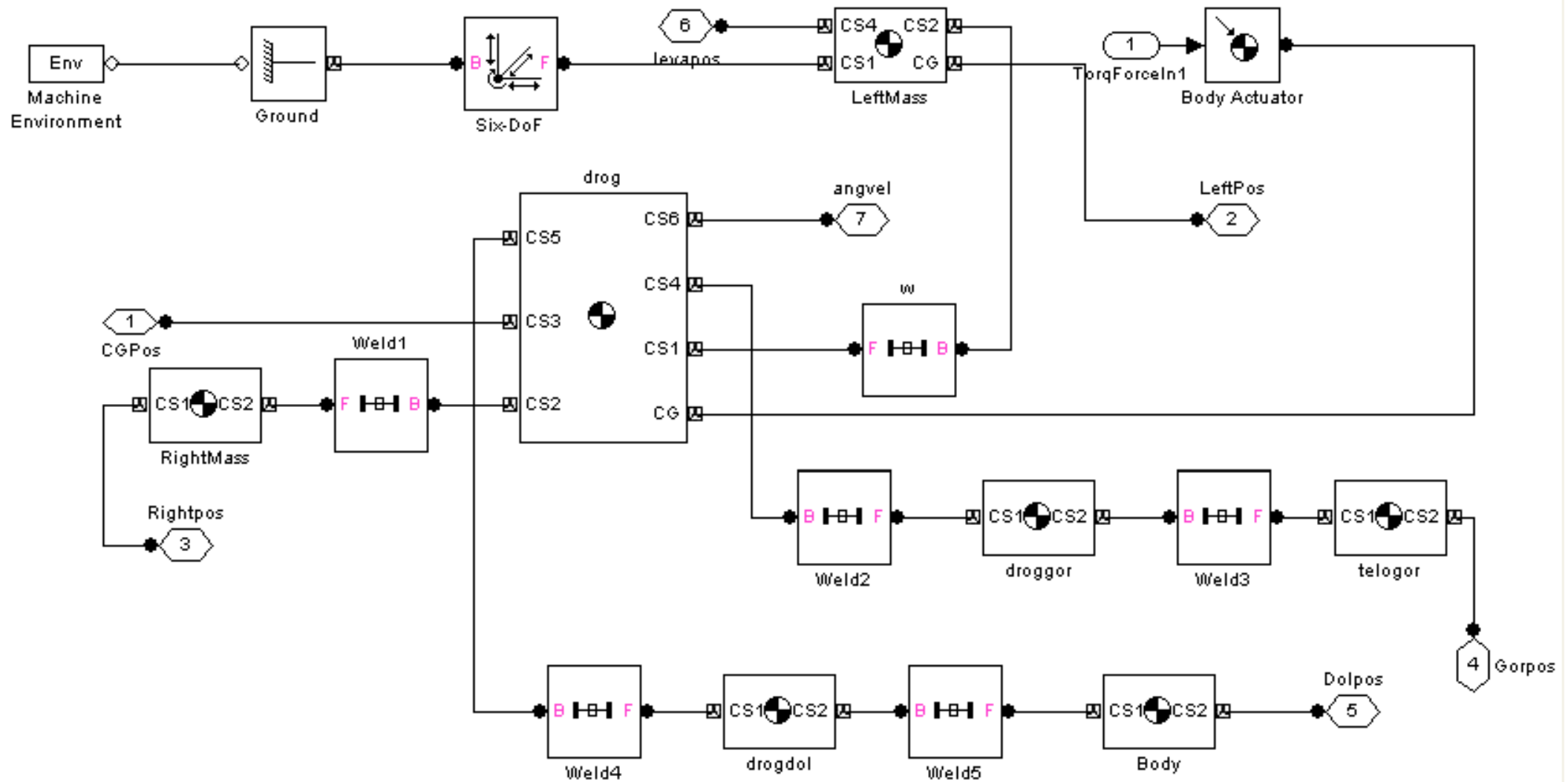


Joint Actuator

Test gibanja trdnega telesa



Model ravninske dvojne ročke



'MATLAB' solver

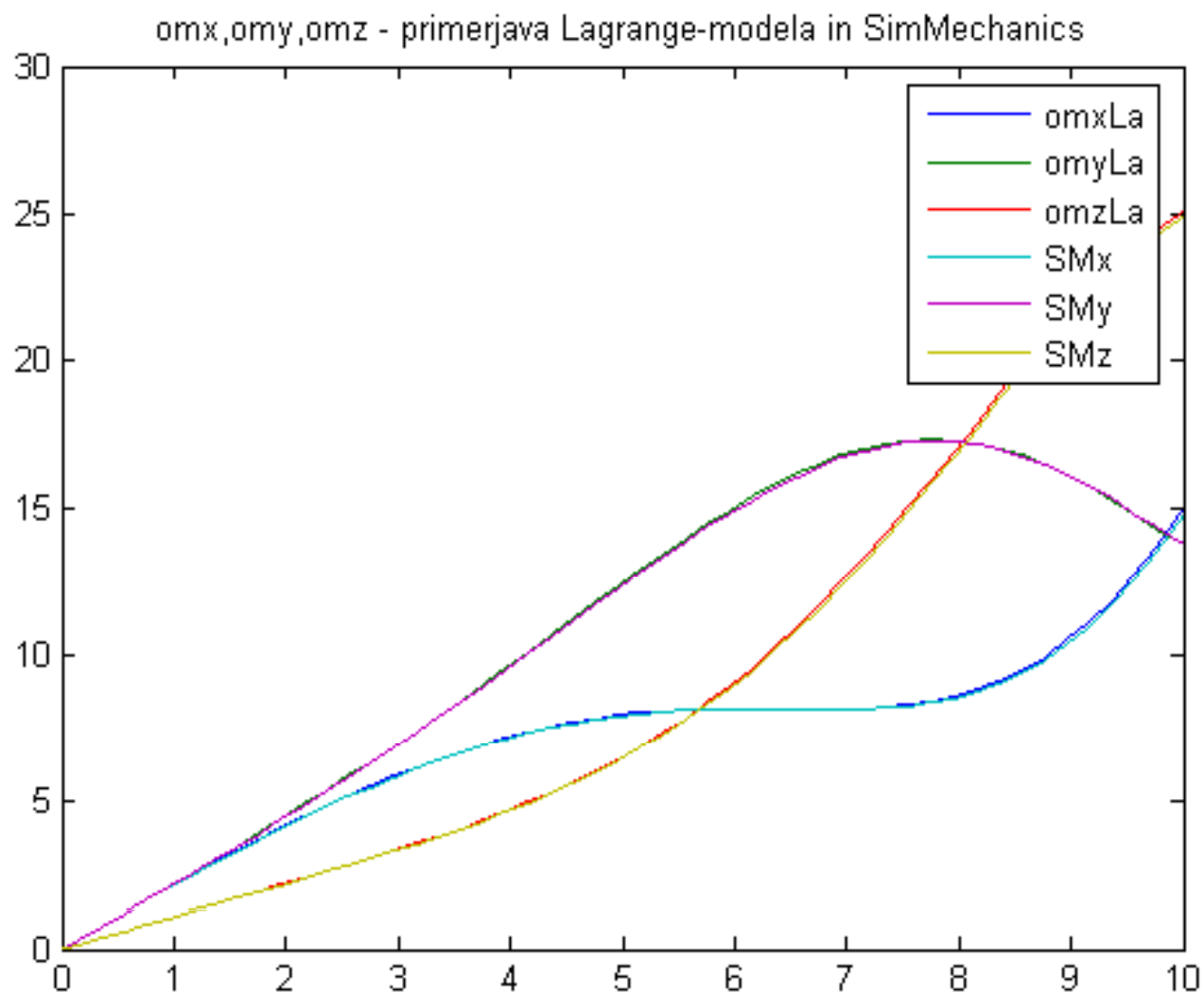
$$\frac{d}{dt} \left[\sum r_i \times m_i (\underline{\omega} \times r_i) \right] = \underline{T} \quad T = \underline{\omega} \times I \cdot \underline{\omega} + I \frac{d\underline{\omega}}{dt}$$

$$I = \begin{pmatrix} y^2 + z^2 & -xy & -xz \\ -xy & x^2 + z^2 & -yz \\ -xz & -yz & x^2 + y^2 \end{pmatrix}; \text{ za točko : } (x, y, z)^T$$

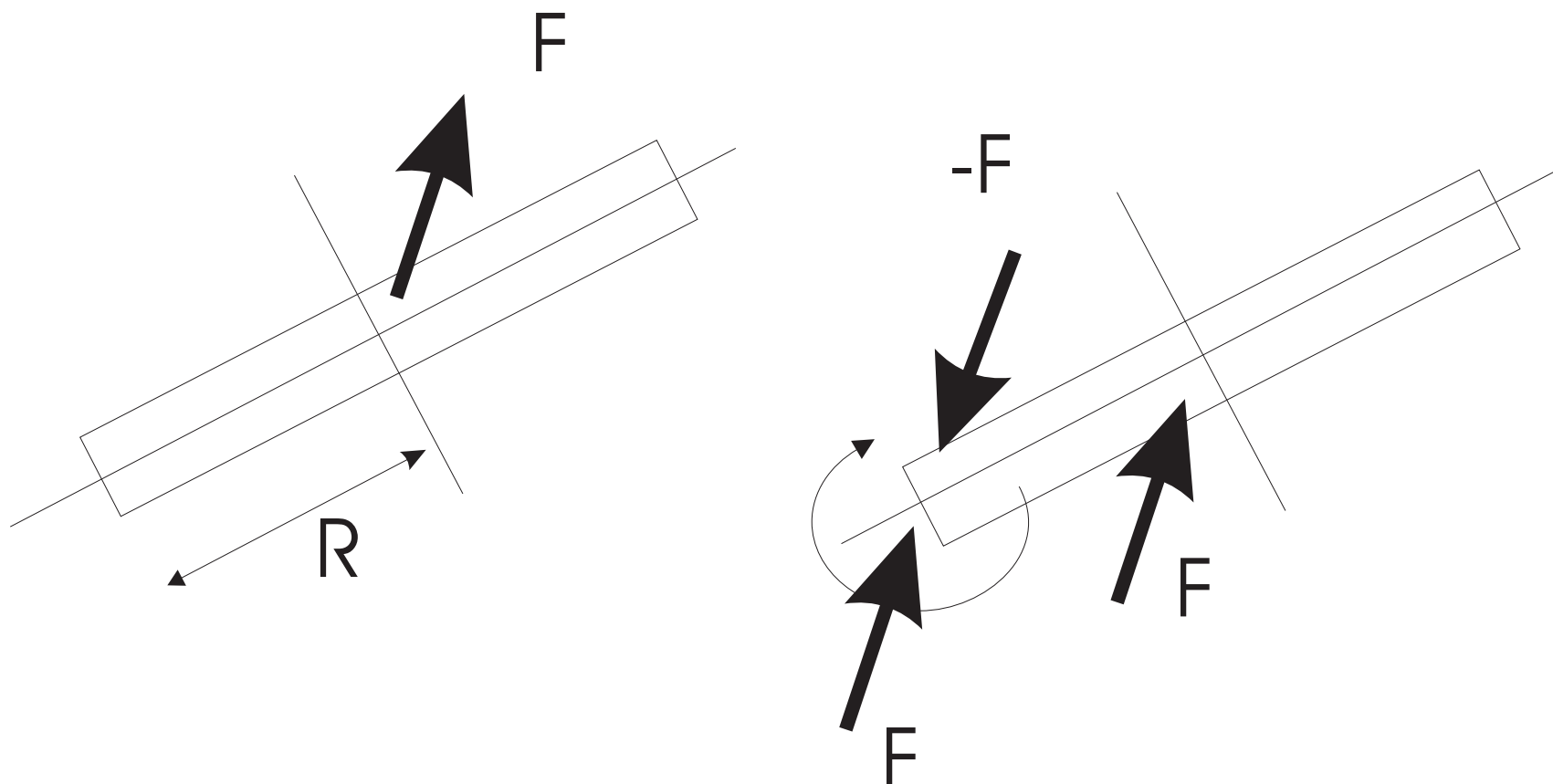
vazno : rotacijska matrika A : $I_0 = A^T \cdot I \cdot A$

[vect, omri] = ode23 (@fDeromThRpikaR, vect,
[omini, thini, r1pikaini, r2pikaini, r3pikaini, r4pikaini,...
R1ini,r2ini,r3ini,r4ini]', options) ;

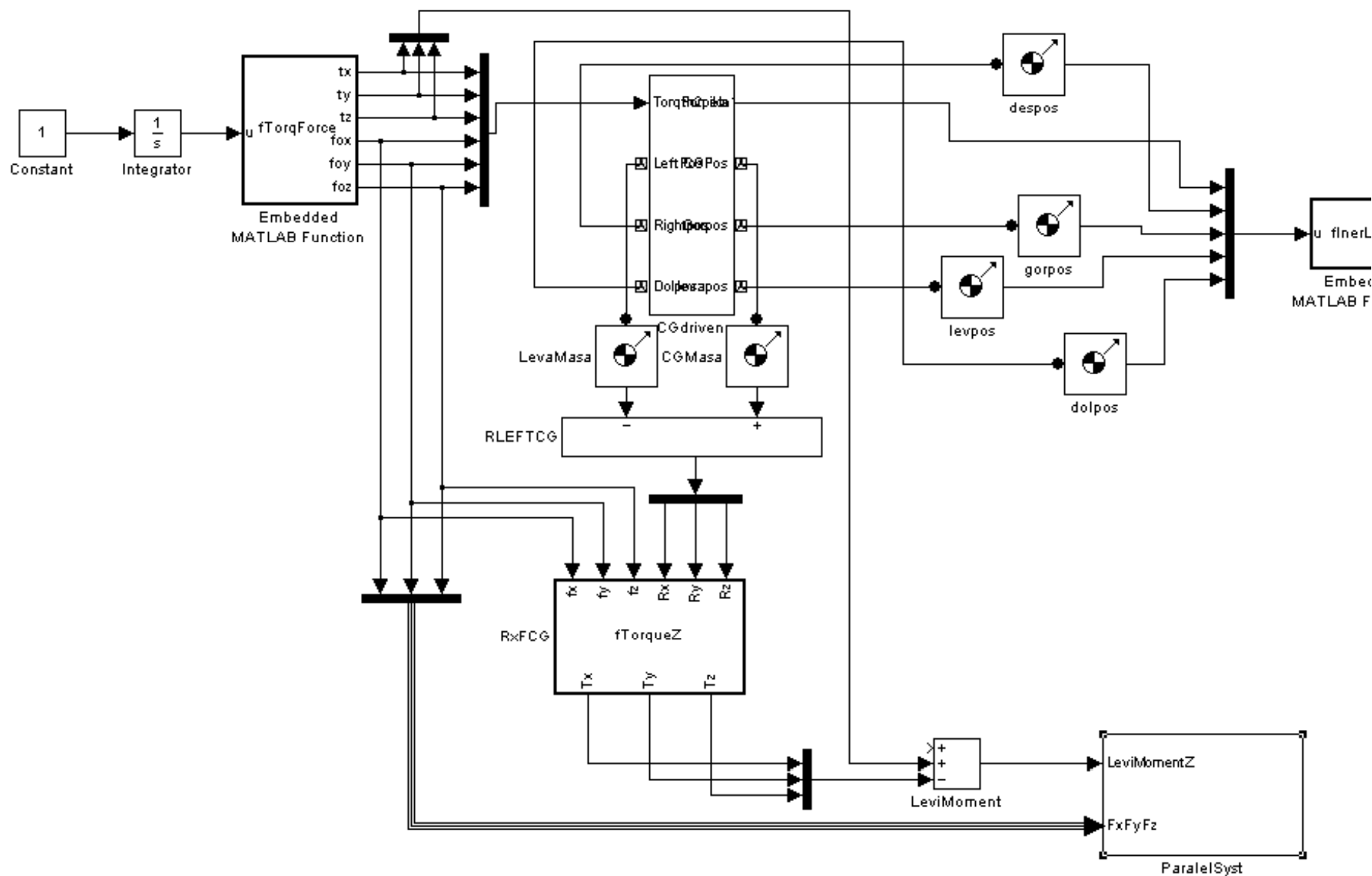
Grafična primerjava rešitev:



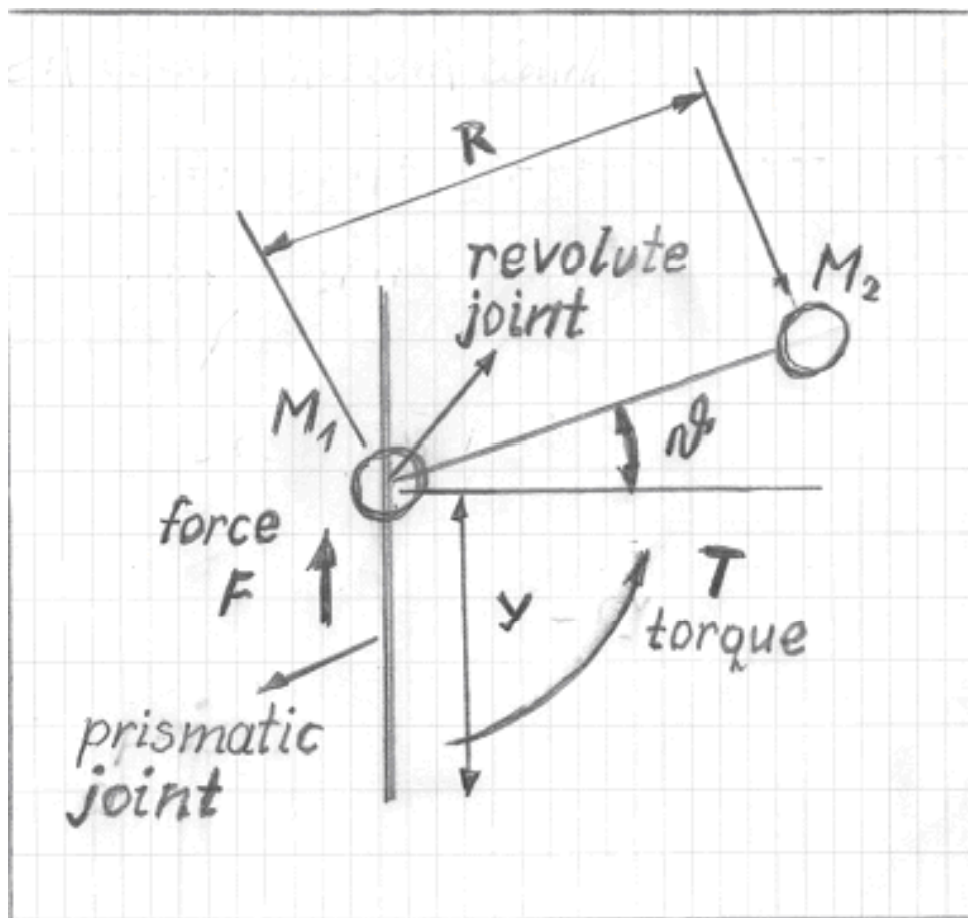
Ekvivalenten sistem sil in momentov :



Ekvivalenten sistem sil in momentov :



Kinematska veriga: prizma + rotacija : shema



Kinematska veriga: prizma + rotacija : Lagrange

$$v_L = [0, \dot{y}, 0]; \quad \omega_L = [0, 0, \dot{\vartheta}]; \quad \rho_D = [R \cos(\vartheta); R \sin(\vartheta); 0]$$

$$v_D = v_L + \omega_L \times \rho_D$$

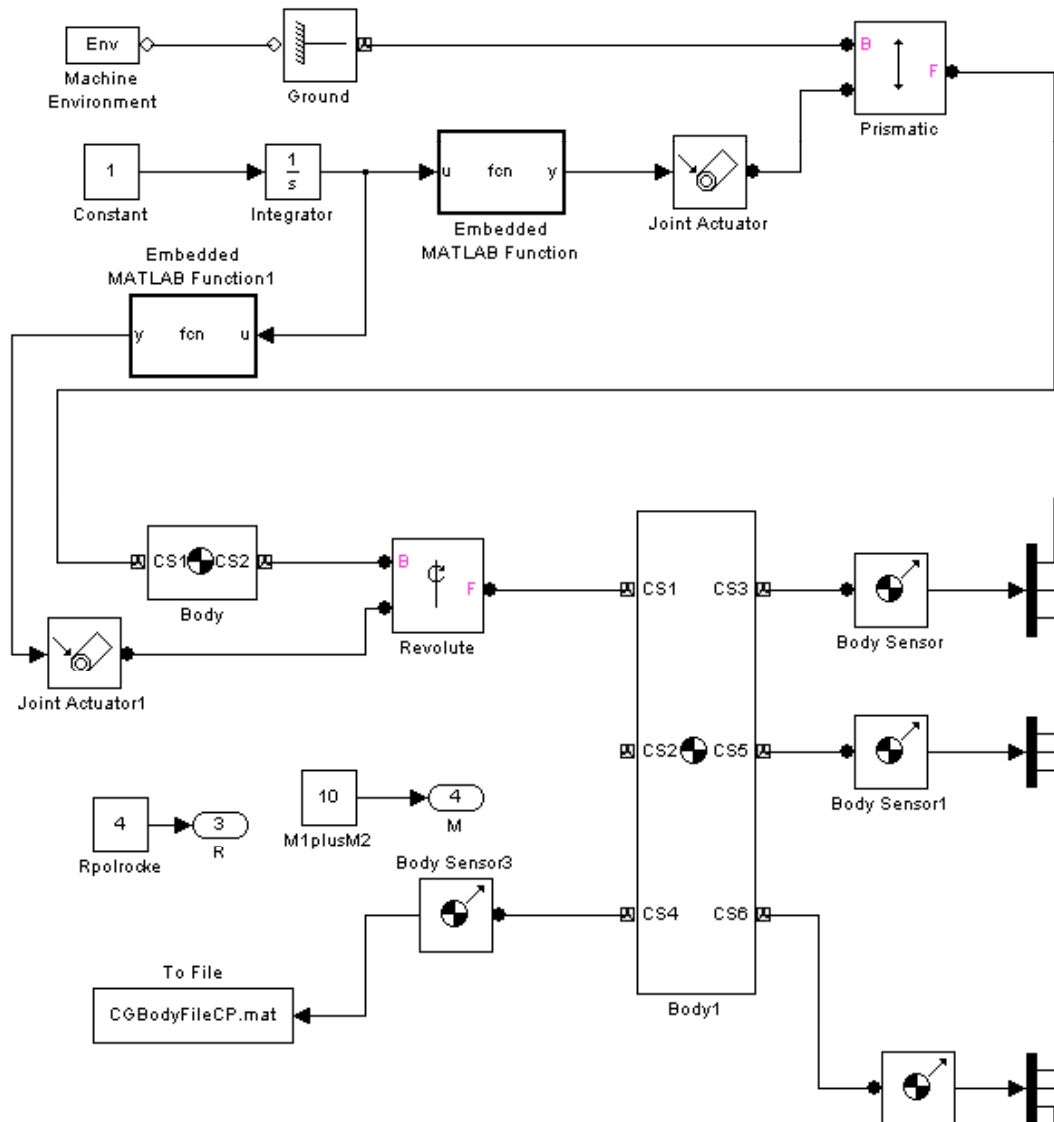
$$KE = \frac{1}{2} m v_L^2 + \frac{1}{2} m v_D^2 \quad L = m \dot{y}^2 + \frac{1}{2} m R^2 \dot{\vartheta}^2 + m R \cos(\vartheta) \dot{y} \dot{\vartheta}$$

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = Q_i$$

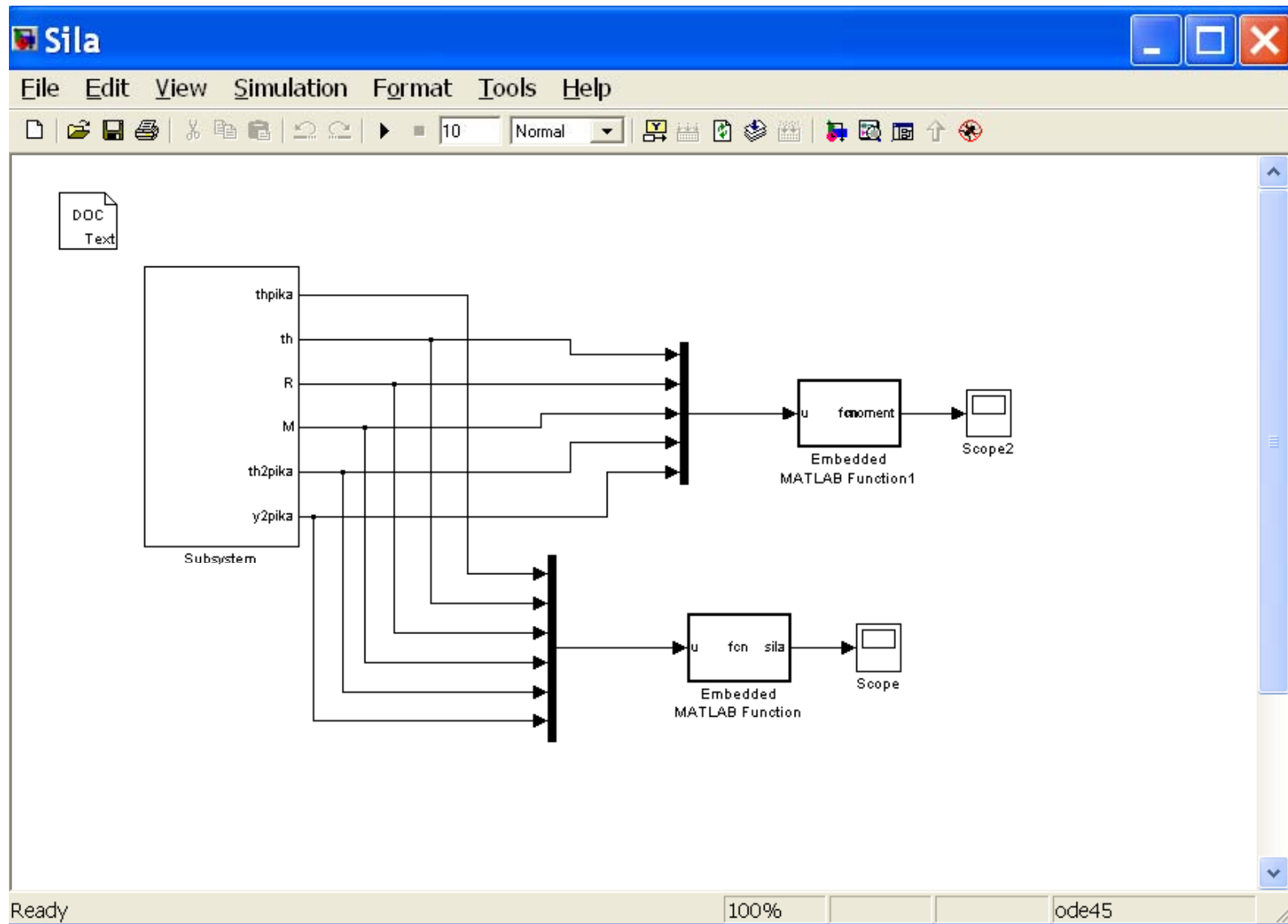
$$2m\ddot{y} + mR \cos(\vartheta) \ddot{\vartheta} - mR \sin(\vartheta) \dot{\vartheta}^2 = F$$

$$mR^2 \ddot{\vartheta} + mR \cos(\vartheta) \ddot{y} = T$$

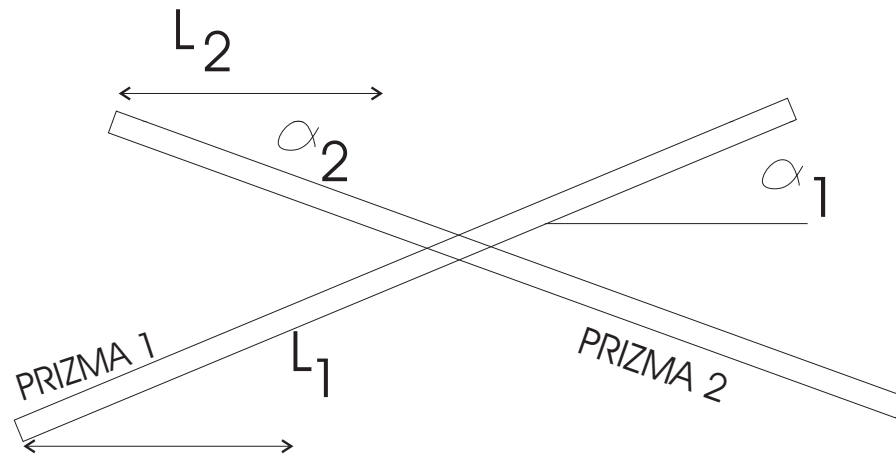
Kinematska veriga: prizma + rotacija : model



Kinematska veriga: prizma + rotacija : inverzne sile



Prizmatska veriga: shema



Lagrangeove enačbe

$$m_1 \ddot{l}_1 + m_2 \ddot{l}_1 + m_2 \ddot{l}_2 C_2 = F_1$$

$$m_2 \ddot{l}_2 + m_2 \ddot{l}_1 C_2 = F_2$$

$$KE = \frac{1}{2} m_1 \dot{l}_1^2 + \frac{1}{2} m_2 (\dot{l}_1^2 + \dot{l}_2^2) + m_2 \dot{l}_1 \dot{l}_2 C_2; C_2 = \sin(\alpha_1) \sin(\alpha_2) + \cos(\alpha_1) \cos(\alpha_2)$$

Rekurzija

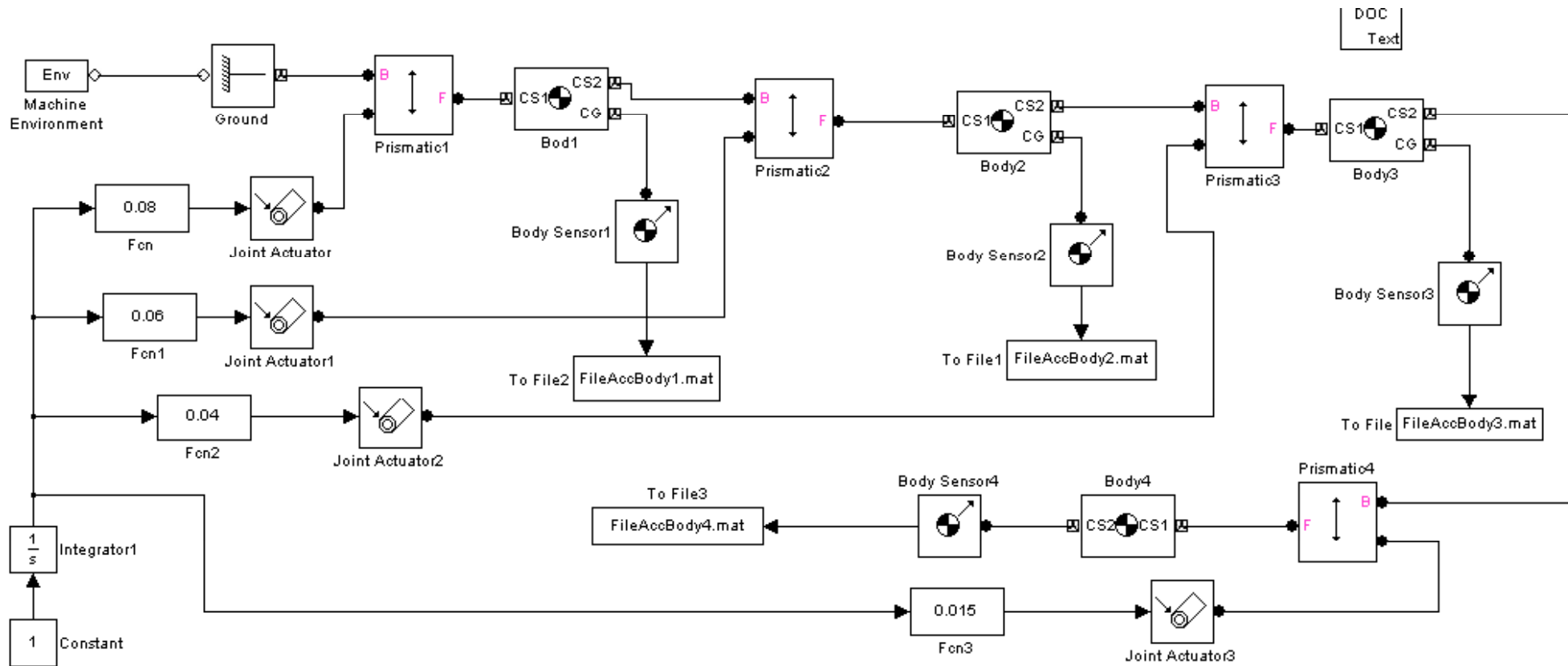
$$\ddot{l}_1 = \frac{C_2 F_2 - F_1}{m_1 + m_2 - m_2 C_2^2}$$

$$F_i = m_i \ddot{r}_i c_i;$$

$$\ddot{l}_2 = \frac{-m_2 C_2 F_1 + F_2 m_1 + F_2 m_2}{m_2 (m_1 + m_2 - m_2 C_2^2)}$$

$$F_{i-1} = \left(m_{i-1} \ddot{r}_{i-1} + \sum_{j=i-1}^n m_j \ddot{r}_j \right) \cdot c_j; \quad i = n, n-1, \dots, 1$$

Prizmatska veriga: inverzne sile: SimMechanics model



Inverzna dinamika splošne kinematske verige:

N - ti člen manipulatorja: $\mathcal{G}_n, \dot{\mathcal{G}}_n, \ddot{\mathcal{G}}_n, I_n = 2m_n R_n^2$

Termi v n-tem členu manipulatorja:

$\tau_{I,CG,n} = 2R_n^2 m_n \ddot{\mathcal{G}}_n$...inercialni moment radi kotnega pospeška

$F_{CG,n} = 2m_n \ddot{r}_{CG,n}$...sila v težišču

$R_{CG,n} = (R_n \cos \mathcal{G}_n; R_n \sin \mathcal{G}_n; 0)$... člen kot vektor

$\tau_{L,n} = -F_{CG,n} \times R_{CG,n}$... prenos momenta na levo

$\tau_{n,gen} = \tau_{L,n} + \tau_{I,CG,n}$... generalizirani externi moment

Inverzna dinamika: rekurzija (od četrtega na prvi člen)

$$\tau_{I,CG,1} = 2R_1^2 m_1 \ddot{\mathcal{G}}_1 \quad \text{inercialni moment zaradi kotnega pospeška}$$

$$F_{CG,1} = 2m_1 \ddot{r}_{CG,1} \quad \text{sila v težišču}$$

$$R_{CG,1} = (R_1 \cos \mathcal{G}_1; R_1 \sin \mathcal{G}_1; 0) \quad \text{člen kot vektor}$$

$$\tau_{L,1} = -F_{CG,1} \times R_{CG,1} \quad \text{prenos momenta na levo}$$

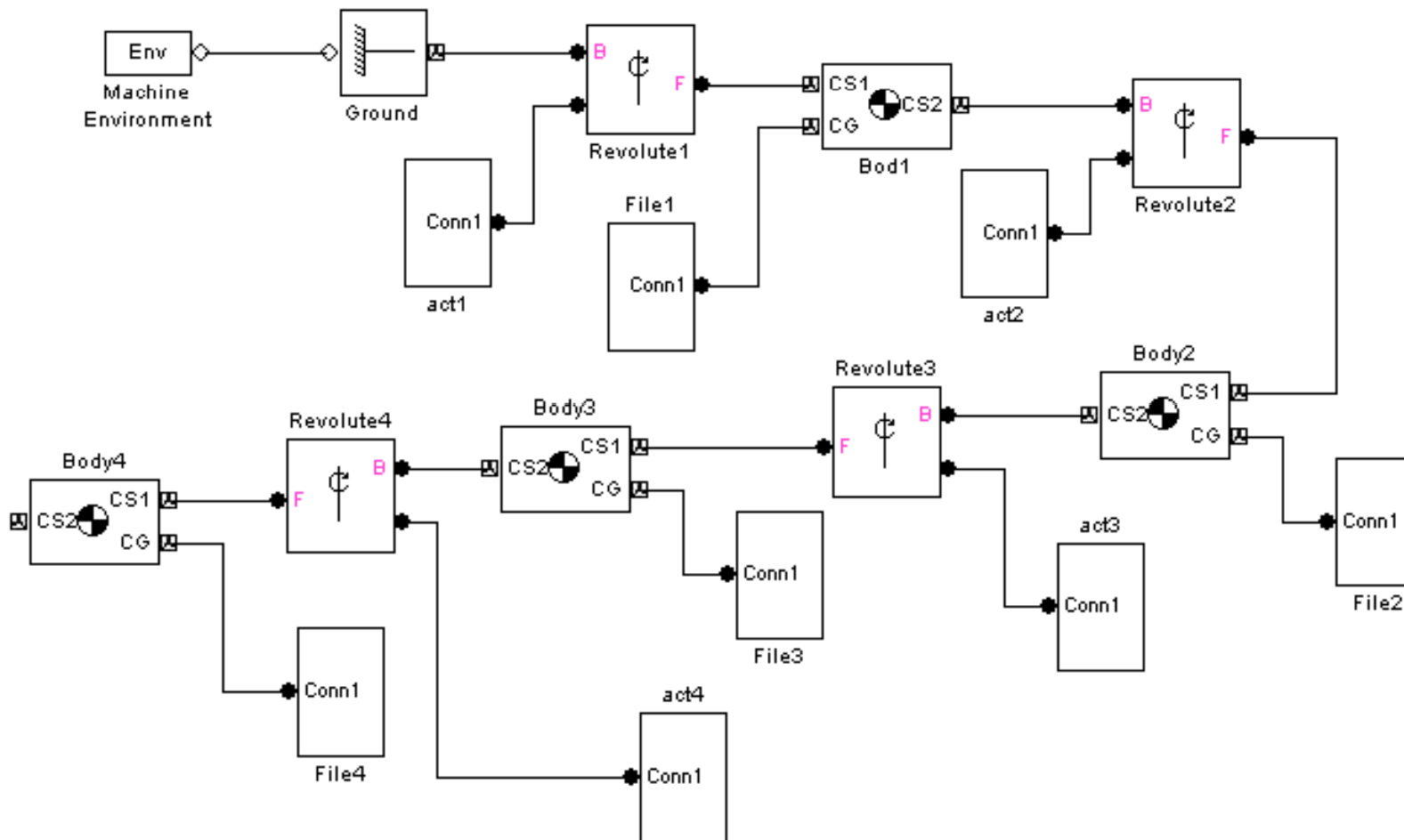
$$\tau_{F4,L1} = (2R_{CG,1} + 2R_{CG,2} + 2R_{CG,3}) \times F_{CG,4} \quad \text{moment sile iz čl.4}$$

$$\tau_{F3,L1} = (2R_{CG,1} + 2R_{CG,2}) \times F_{CG,3} \quad \text{moment sile iz čl.3}$$

$$\tau_{F2,L1} = (2R_{CG,1}) \times F_{CG,2} \quad \text{moment sile iz čl.2}$$

$$\begin{aligned} \tau_{L1,gen} = & \tau_{L,4} + \tau_{L,3} + \tau_{L,2} + \tau_{L,1} + \\ & + \tau_{I,CG,4} + \tau_{I,CG,3} + \tau_{I,CG,2} + \tau_{I,CG,1} + \\ & + \tau_{F4,L1} + \tau_{F3,L1} + \tau_{F2,L1} \end{aligned}$$

Inverzna dinamika: Simulink model, 4-členi manipulator



ZAKLJUČEK

Prispevek predstavlja osnovne probleme iz dinamike in inverzne dinamike robotskih manipulatorjev. Bolj kot posamezne rešitve preprostih primerov je v center postavljena uporaba programskih orodij, kot je MATLAB v kombinaciji s poddomenami SIMULINK in SimMechanics, temu bi lahko pridružili še STATEFLOW, s katerim je mogoče vnesti principe odločanja in logičnega programiranja. Seveda gre za obravnavo znanih stvari, zato tudi v primerih, ko posamezni problemi niso bili direktno najdeni v literaturi, rešitve nekje zagotovo obstajajo. Zato tudi v bibliografiji niso navedena posamezna dela, ki vsebujejo ravno take – ali vsaj zelo podobne primere – temveč so v tem pregledu navedena dela, ki nekako predstavljajo osnovo za nadaljno spoznavanje te problematike. Niti v samem prispevku bibliografski citati niso navedeni, saj jih je možno najti v večini del iz spodaj navedenih virov.